HARMONIC-BALANCE SIMULATION OF STRONGLY NONLINEAR VERY LARGE-SIZE MICROWAVE CIRCUITS BY INEXACT NEWTON METHODS

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ABSTRACT

The paper introduces a new approach to harmonic-balance simulation, based on inexact Newton methods and iterative system-solving techniques. Storage and factorization of the Jacobian matrix are avoided, resulting in a dramatic drop of execution time and memory occupation. HB analyses with several tens of thousands unknowns become possible on ordinary workstations.

INTRODUCTION

State-of-the-art harmonic-balance (HB) simulators for the steady-state analysis of nonlinear microwave circuits usually rely upon the Newton method coupled with some sort of globalizing mechanism such as norm reduction [1]. Indeed, it has been found that norm-reducing Newton methods based on the exact computation of the Jacobian matrix and on a suitable parametric formulation of the strongest device nonlinearities, may provide very robust solution algorithms that can efficiently handle forcing amplitudes of hundreds of kilovolts [2]. This is not only true for the usual devices normally encountered in microwave circuits, but also for some kinds of very ill-conditioned nonlinearities that may occur in power circuits, such as ferromagnetic hysteresis [3]. Thus the only outstanding drawback of this simulation technique is the rapidly increasing size of the numerical problem as the number of active devices and/or of spectral lines becomes large. This issue is increasingly important for modern microwave applications, since integrated circuit topologies and signal spectra tend to become more and more complex (e.g., in cellular systems applications). For this reason, the problem of large-size HB simulations has been tackled by several authors in the recent technical literature. The proposed approaches are usually aimed at avoiding the storage and factorization of the full Jacobian matrix. In [4], [5] this is obtained by solving the linear system at each step of the Newton algorithm by an iterative technique with suitable preconditioning. In [2], [6] a large percentage of the entries of the Jacobian matrix is artificially set to 0 according to a suitable numerical or physical criterion, and a sparse-matrix solver is used for the linear system. Generally speaking, the penalty for the use of such techniques is a considerable limitation of the power-handling capabilities of the HB simulator. As an example, ref. [2] reports the intermodulation (IM) analysis of a distributed DGFET mixer requiring 3600 unknowns. Convergence is achieved by the sparse-matrix technique up to an LO drive of about 3 V. As a further example, ref. [5] reports the harmonic distortion analysis of an MOS amplifier requiring 5832 unknowns. Convergence is achieved by the iterative technique up to a drive level of about 0.4 V.

The paper discusses a novel approach to the solution of large-size HB simulation problems, which is very well suited for strongly nonlinear applications. The analysis relies on an inexact Newton method, and uses an iterative technique to approximately solve the Newton equation, thus avoiding the storage and factorization of the Jacobian matrix. High numerical efficiency is obtained by suitably choosing the accuracy requirements for the approximate solution at each step. Excellent robustness is provided by a norm-reduction scheme. The capabilities of the new technique are demonstrated by the gain compression and intermodulation analysis of a strongly nonlinear circuit, consisting of a double-ring diode mixer with active baluns. In particular, a large-signal 5-th order IM analysis requiring 33184 unknowns - probably the largest HB simulation ever reported - is demonstrated on an ordinary workstation.

The new approach retains all the capabilities and the advantages of multiple FFT-based HB techniques [2], is faster than traditional HB algorithms, and effectively overcomes the size problem for large circuits. Its results are numerically exact, which is of primary importance in the evaluation of critical performance aspects such as high-order IM products. This method is thus believed to represent a real breakthrough in numerical simulation techniques for analog nonlinear circuits.

TH 2B

INEXACT NEWTON METHODS

Let us consider the nonlinear system of N equations in N unknowns

$$\mathbf{E}(\mathbf{X}) = \mathbf{0} \tag{1}$$

where $\mathbf{E} : R^N \rightarrow R^N$ is continuously differentiable. The *exact* Newton method for the solution of (1) is the computation of a sequence of iterates $\mathbf{X}_i$ by means of the recursive relation

$$\begin{aligned} &\mathbf{J}(\mathbf{X}_i)\,\mathbf{n}_i = -\mathbf{E}(\mathbf{X}_i) \\ &\mathbf{X}_{i+1} = \mathbf{X}_i + \mathbf{n}_i \end{aligned} \tag{2}$$

where $\mathbf{n}_i$ represents the exact Newton update and $\mathbf{J}(\mathbf{X})$ is the N x N Jacobian matrix of the system (1) ($\mathbf{J}(\mathbf{X}) = \partial\mathbf{E}/\partial\mathbf{X}$). The exact solution of the Newton equation (i.e., the first of (2)) is expensive when N is large, and may not be worthwhile in general especially at the first steps. A more efficient solution approach may be developed by resorting to an *inexact* Newton method [7]. In this case a sequence of iterates is generated by

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finding at each step some update $\mathbf{s}_i$ and some *forcing term* f_i ($0 \le f_i < 1$) such that

$$\| \mathbf{E}(\mathbf{X}_i) + \mathbf{J}(\mathbf{X}_i)\, \mathbf{s}_i \| \le f_i \| \mathbf{E}(\mathbf{X}_i) \|$$
$$\mathbf{X}_{i+1} = \mathbf{X}_i + \mathbf{s}_i \qquad (3)$$

according to a suitable definition of the norm $\| \bullet \|$. For $f_i = 0$ (3) reduces to (2); otherwise, the forcing term is in some way a measure of the allowed deviation of the current update from the Newton update defined by (2).

In the practical use of (3), the forcing term is first chosen at each step making use of a suitable algorithm. For HB analysis applications, excellent results have been obtained by means of the update formula [8]

$$f_i = \frac{\| \mathbf{E}(\mathbf{X}_i) - \mathbf{E}(\mathbf{X}_{i-1}) - \mathbf{J}(\mathbf{X}_{i-1})\, \mathbf{s}_{i-1} \|}{\| \mathbf{E}(\mathbf{X}_{i-1}) \|} \qquad (4)$$

with $f_0 = 0.5$. Note that (4) is a normalized measure of the deviation between the nonlinear map $\mathbf{E}(\mathbf{X})$ and its local linear model at the (i - 1)-th step. After choosing the forcing term, the Newton equation is approximately solved in such a way that the approximate solution $\mathbf{s}_i$ satisfies the first of (3) for the given f_i.

One effective way of doing so is to make use of an iterative method, whereby the accuracy of the solution is refined step by step until the first of (3) is met. This usually provides an excellent tradeoff between accuracy and CPU time consumption. In addition, iterative methods do not require the storage and factorization of the Jacobian matrix, but only the repeated computation of matrix-vector products of the form $\mathbf{J}(\mathbf{X})\,\mathbf{z}$, where $\mathbf{z}$ generically denotes a vector of size N. This is obviously a key advantage if N is very large. On the other hand, if N is so large that the Jacobian matrix cannot be stored, the entries of $\mathbf{J}(\mathbf{X})$ must be cyclically re-computed as needed in order to generate the products $\mathbf{J}(\mathbf{X})\,\mathbf{z}$. As we shall see in the next section, this potential drawback can be nicely overcome when (1) is the nonlinear solving system for a harmonic-balance analysis, thanks to the peculiar structure of the Jacobian matrix. Finally, it is worth noting that the approximation of the Jacobian matrix does not affect the accuracy of the results: whenever convergence is achieved, the solution of (1) is *exact* [7]. The convergence properties of an iterative solver can be considerably improved by suitably *preconditioning* the Newton equation, i.e., replacing the first of (2) by

$$\mathbf{J}(\mathbf{X}_i)\, \mathbf{P}_i^{-1}\, \mathbf{P}_i\, \mathbf{n}_i = -\, \mathbf{E}(\mathbf{X}_i) \qquad (5)$$

where $\mathbf{P}_i$ is a nonsingular N x N matrix [9]. Intuitively, $\mathbf{P}_i$ should be chosen is such a way that the preconditioned system matrix $\mathbf{J}(\mathbf{X}_i)\, \mathbf{P}_i^{-1}$ be as close as possible to an identity matrix, compatibly with the constraint that the computation of the product $\mathbf{P}_i^{-1}\,\mathbf{z}$ should have negligible cost in comparison with the computation of $\mathbf{J}(\mathbf{X}_i)\,\mathbf{z}$.

APPLICATION TO HARMONIC-BALANCE ANALYSIS

Following [2], we describe the nonlinear subnetwork by a set of generalized parametric equations of the form

$$\mathbf{v}(t) = \mathbf{u}\left[\mathbf{x}(t),\ \frac{d\mathbf{x}}{dt},\ \ldots,\ \frac{d^n\mathbf{x}}{dt^n},\ \mathbf{x}_d(t)\right]$$
$$\mathbf{i}(t) = \mathbf{w}\left[\mathbf{x}(t),\ \frac{d\mathbf{x}}{dt},\ \ldots,\ \frac{d^n\mathbf{x}}{dt^n},\ \mathbf{x}_d(t)\right] \qquad (6)$$

In (6) $\mathbf{x}(t)$ is a vector of state variables (SV), $\mathbf{v}(t)$, $\mathbf{i}(t)$ are vectors of voltages and currents at the device ports, and $\mathbf{x}_d(t)$ is a vector of time-delayed state variables, i.e., $x_{di}(t) = x_i(t - \tau_i)$ where the τ_i's are constant delays. The vector-valued functions $\mathbf{u}$, $\mathbf{w}$ are assumed to be nonlinear and memoryless. All vectors have the same size n_D, equal to the number of device ports. In the general case of multitone excitation, a generic IM product of the exciting tones is identified by a vector $\mathbf{k} = [k_1, k_2, \ldots]^T$ of harmonic numbers, and is denoted by $\Omega_\mathbf{k}$. Let $\mathbf{U}_\mathbf{k}$, $\mathbf{W}_\mathbf{k}$ be the vectors containing the $\mathbf{k}$-th harmonics of the voltage and current harmonics. The vector of (complex) harmonic-balance errors at $\Omega_\mathbf{k}$ is then [2]

$$\mathbf{E}_\mathbf{k} = \mathbf{Y}(\Omega_\mathbf{k})\, \mathbf{U}_\mathbf{k} + \mathbf{W}_\mathbf{k} + \mathbf{N}_\mathbf{k} \qquad (7)$$

where $\mathbf{Y}(\omega) = \mathbf{G}(\omega) + j\mathbf{B}(\omega)$ is the linear subnetwork admittance matrix, and $\mathbf{N}_\mathbf{k}$ is the vector of Norton equivalent current sources of the free generators of frequency $\Omega_\mathbf{k}$.

In practice, in order to avoid the use of negative frequencies, the circuit equations are formulated in terms of real and imaginary parts of both the HB errors and the SV harmonics. Thus $\mathrm{Re}[\mathbf{E}_\mathbf{k}]$ and $\mathrm{Im}[\mathbf{E}_\mathbf{k}]$ (for all $\mathbf{k}$'s) are stacked to form a real error vector $\mathbf{E}$, and so are $\mathrm{Re}[\mathbf{X}_\mathbf{k}]$ and $\mathrm{Im}[\mathbf{X}_\mathbf{k}]$ to form a vector $\mathbf{X}$ of real unknowns. The nonlinear solving system is then written in the form (1), and its size is $N = n_D(2n_H + 1)$ where n_H is the number of spectral lines (not including d.c.). Accordingly, the Jacobian matrix $\mathbf{J}(\mathbf{X})$ is partitioned frequency-wise into submatrices $\mathbf{J}_{\mathbf{k},\mathbf{s}}$ of the form

$$\mathbf{J}_{\mathbf{k},\mathbf{s}} = \begin{bmatrix} \dfrac{\partial \mathrm{Re}[\mathbf{E}_\mathbf{k}]}{\partial \mathrm{Re}[\mathbf{X}_\mathbf{s}]} & \dfrac{\partial \mathrm{Re}[\mathbf{E}_\mathbf{k}]}{\partial \mathrm{Im}[\mathbf{X}_\mathbf{s}]} \\ \dfrac{\partial \mathrm{Im}[\mathbf{E}_\mathbf{k}]}{\partial \mathrm{Re}[\mathbf{X}_\mathbf{s}]} & \dfrac{\partial \mathrm{Im}[\mathbf{E}_\mathbf{k}]}{\partial \mathrm{Im}[\mathbf{X}_\mathbf{s}]} \end{bmatrix} \qquad (8)$$

From (7) we obtain directly

$$\frac{\partial \mathrm{Re}[\mathbf{E}_\mathbf{k}]}{\partial \mathrm{Re}[\mathbf{X}_\mathbf{s}]} = \mathbf{G}(\Omega_\mathbf{k}) \frac{\partial \mathrm{Re}[\mathbf{U}_\mathbf{k}]}{\partial \mathrm{Re}[\mathbf{X}_\mathbf{s}]} - \mathbf{B}(\Omega_\mathbf{k}) \frac{\partial \mathrm{Im}[\mathbf{U}_\mathbf{k}]}{\partial \mathrm{Re}[\mathbf{X}_\mathbf{s}]} + \frac{\partial \mathrm{Re}[\mathbf{W}_\mathbf{k}]}{\partial \mathrm{Re}[\mathbf{X}_\mathbf{s}]} \qquad (9)$$

and the like. The derivatives of the voltage and currents harmonics $\mathbf{U}_\mathbf{k}$, $\mathbf{W}_\mathbf{k}$ may be found in the following way. We first introduce the Fourier expansions

$$\frac{\partial \mathbf{u}}{\partial y_m} = \sum_\mathbf{p} \mathbf{C}_{m,\mathbf{p}} \exp(j\Omega_\mathbf{p} t)$$
$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}_\mathbf{d}} = \sum_\mathbf{p} \mathbf{C}_\mathbf{p}^\mathbf{d} \exp(j\Omega_\mathbf{p} t)$$
$$\frac{\partial \mathbf{w}}{\partial y_m} = \sum_\mathbf{p} \mathbf{D}_{m,\mathbf{p}} \exp(j\Omega_\mathbf{p} t) \qquad (10)$$
$$\frac{\partial \mathbf{w}}{\partial \mathbf{x}_\mathbf{d}} = \sum_\mathbf{p} \mathbf{D}_\mathbf{p}^\mathbf{d} \exp(j\Omega_\mathbf{p} t)$$

where $\mathbf{y}_0 = \mathbf{x}$, $\mathbf{y}_m = d^m\mathbf{x}/dt^m$ ($1 \le m \le n$). The derivatives of $\mathbf{U}_k$, $\mathbf{W}_k$ with respect to the real and imaginary parts of the SV harmonics are then linear combinations of the Fourier coefficients $\mathbf{C}_{m,k\pm s}$, $\mathbf{D}_{m,k\pm s}$, $\mathbf{C}^d_{k\pm s}$, $\mathbf{D}^d_{k\pm s}$, whose explicit expressions may be found in [2]. Note that the **C**, **D** coefficients are formally complex matrices of size $n_D \times n_D$. However, the nonlinear subnetwork is usually a set of uncoupled devices (such as diodes or transistors), each having a small number of ports (usually 1 or 2). Thus the **C**, **D** matrices are in reality block diagonal, with very small diagonal blocks. Also, some of the entries of the **C**, **D** matrices may be identically zero depending on the device models. If we denote by M_i the total number of nonzero scalar Fourier coefficients appearing in the expansions (10) associated with the i-th device, the memory occupation of the **C**, **D** matrices expressed in memory words is

$$M_{CD} = 2\, n_H\, 2^T \sum_i M_i \qquad (11)$$

where T is the number of intermodulating tones, and the summation is extended over all nonlinear devices. This is usually well within the reach of any ordinary workstation, as shown by the examples reported in the following section.

The above-discussed structure of the Jacobian matrix lends itself nicely to the use of an iterative method in the solution of the Newton equation. The linear subnetwork admittance is computed and stored once for all at the beginning of the analysis. The Fourier expansions (10) are computed by the multidimensional FFT and stored at the beginning of each Newton step. Thus all the information needed in order to multiply the Jacobian matrix by a vector is available in memory, and need not be re-computed at each iteration, in spite of the fact that the entire Jacobian matrix is not pre-computed and stored. In this way the solution process becomes very efficient. A further considerable advantage of this approach is that it makes naturally available an effective way of preconditioning the Newton equation. It turns out that the artificially sparse Jacobian matrix discussed in [2] represents an excellent preconditioner for broad classes of microwave circuits. Indeed, it is easy to find an intuitive explanation of this result: the sparse Jacobian matrix is inexpensive to factorize due to its block-diagonal structure, and at the same time is accurate enough to ensure convergence of the sequence of iterates (2) at low and moderate drive levels [2].

In view of the application to the HB analysis of strongly nonlinear microwave circuits, the GMRES method [10] has been selected out of the many iterative techniques reported in the literature, because of its robustness [9]. A norm-reduction mechanism (backtracking) has also been implemented in order to improve the global convergence properties of the inexact Newton method.

NUMERICAL RESULTS AND DISCUSSION

The harmonic-balance analysis technique discussed in the previous sections has been successfully applied to a number of large-size nonlinear simulation problems. As representative examples, we report here on some results obtained for a double-ring balanced mixer with the topology schematically illustrated in fig. 1 [11]. In this circuit the two diode rings are the mixing elements, and the distributed FET structures act as broadband baluns at the RF, LO, and IF ports. The use of active baluns makes the mixer suitable for monolithic integration. The circuit contains 8 diodes and 12 FET's for a total of $n_D = 32$ device ports. The diodes are driven into forward conduction by the LO pump, and are thus strongly nonlinear due to the exponential behavior of the junction current and of the diffusion capacitance. In all cases the HB analysis was requested to achieve a relative error less than 10^{-3} on each spectral component.

As a first simulation problem, we consider the gain compression analysis of the mixer with 8 local-oscillator harmonics, and 3 upper and 3 lower sidebands per LO harmonic. This problem was already treated in [12] by a hierarchical HB technique, and is thus useful for the sake of comparison. In this case we have $n_H = 59$, so that the analysis requires N = 3808 scalar unknowns. The storage of the **C**, **D** matrices requires $M_{CD} \approx 77$ kwords, corresponding to 0.62 MB. The analysis is carried out with LO drive of +13 dBm, and RF power ranging from -10 dBm to +13 dBm. The results shown in fig. 2 predict a 1-dB compression point of about +12 dBm, in very good agreement with experimental observations [11]. On a SPARCstation 10 with 112 MB of central memory, the average CPU time per point required by the HB analysis based on the inexact Newton method is about 96 seconds. For comparison, the same analysis takes about 230 seconds per point by the hierarchical approach of [12], and as much as 4700 seconds per point by the conventional HB technique based on the computation and storage of the full Jacobian matrix. The results obtained by the three methods are identical. Note that the power sweep is by no means indispensable to ensure convergence of the algorithm: as an example, the analysis at the 1-dB compression point converges nicely starting from zero harmonics in about 226 seconds. In order to verify the power-handling capabilities of the algorithm, the analysis is repeated at +12 dBm RF power and increasing LO drive. Convergence is achieved by the inexact Newton method up to +30 dBm of LO power, corresponding to an LO signal amplitude of 20 V.

We next consider a large-signal two-tone intermodulation analysis of the same mixer with 8 local-oscillator harmonics, and IM products of the two RF carriers up to the 5th order. In this case we have $n_H = 518$, so that the analysis requires N = 33184 scalar unknowns. With the conventional HB approach the Jacobian matrix would take 8.8 GB of memory, and with the hierarchical technique of [12] the storage of the master Jabobian would require 0.55 GB. Thus neither approach is practically usable (not even by resorting to virtual memory). On the other hand, the storage of the **C**, **D** matrices requires $M_{CD} \approx 1.4$ Mwords, corresponding to 11 MB. Thus the new technique does not pose any significant storage problems. The analysis is carried out with LO drive of +13 dBm, and RF power (per tone) ranging from -10 dBm to +13 dBm (about 4-dB gain compression). The results shown in fig. 3 are very well behaved throughout the power range of interest. On the SPARCstation 10 with 112 MB of central memory, the average CPU time per point required by the HB analysis based on the inexact Newton method is about 2990 seconds.

The potential of the new technique can be appreciated by considering that the same number of unknowns can accomodate the analysis of a 600-transistor front-end with an ordinary mixer spectrum of 13 lines plus dc.

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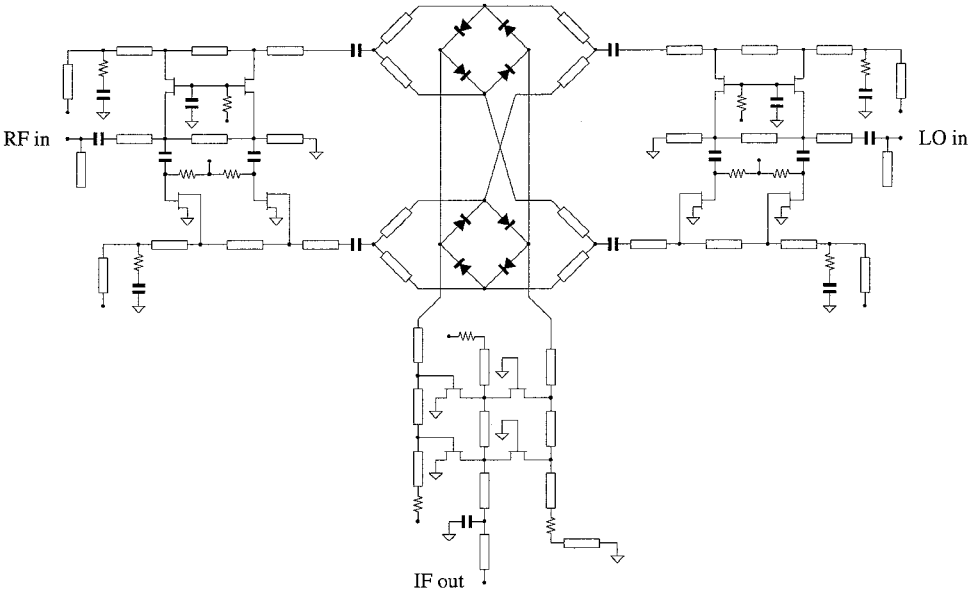

Fig. 1 - Schematic topology of a monolithic double-ring mixer (after [11]).

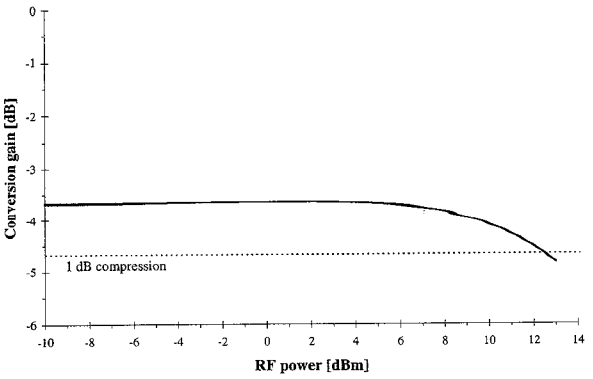

Fig. 2 - Gain compression curve of a monolithic double-ring mixer.

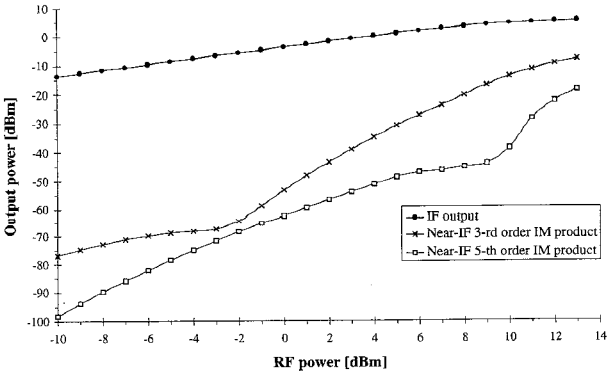

Fig. 3 - Simulated performance of a monolithic double-ring mixer under 2-tone RF excitation.